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Saltation of Fine Particles on Obliquely Oscillating Plate

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Abstract

The saltation of fine particles on an obliquely oscillating plate was simulated using a mass-point model that considered the gravity, fluid resistance, restitution, and friction. To examine the effects of the restitution and fluid resistance on the horizontal transport velocity, the two-dimensional movements of three particles with different coefficients of restitution and diameters were calculated. The results showed that a particle with a higher restitution and smaller diameter had a smaller transport velocity. This can be explained as follows. A particle with a higher restitution bounces forward and backward repeatedly, whereas a particle with a lower restitution only bounces forward, because the higher bounce height increases the impulse exerted on the particle during a collision, which induces a drastic change in the horizontal velocity from the collision. The horizontal velocity of a smaller particle is decreased by the drag force during its flight between successive collisions.

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Keywords: oblique oscillation; fine particle; particle saltation; restitution; fluid resistance

1. Introduction

The kinetic energy of granular materials dissipates as a result of inelastic collisions and the particle–particle and particle–wall friction. The energy input through a vibrating surface induces the fluidization of granules. In physics, the one-dimensional movements of particles bouncing on a vertically oscillating plate have been extensively examined both experimentally and numerically [1-3]. In industry, a vibrating conveyor, in which an oblique

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oscillating trough induces two-dimensional particle movement, is often used for particle transport [4,5]. Although many studies have been conducted on the dynamic behavior of particles on an oscillating plate, the influence of the particle diameter on the behavior has not been systematically examined. In particular, there have been few reports on the behavior of fine particles, which are more useful than coarse particles for many applications because of their large specific surface areas and strong surface activities [6]. A study of the behavior of fine particles may be helpful in their vibration transport.

We previously studied the vertical motion of particle saltation, for a mass median diameter range of 0.5 to 500 μm, on an obliquely oscillating plate [7]. That study showed that restitution of a particle and the fluid resistance, which depends on the particle diameter, have a great effect on the vertical motion. In this paper, to study the influence of the restitution and particle diameter on the horizontal transport velocity, the two-dimensional movements of three particles with different coefficients of restitution and particle diameters on an obliquely oscillating plate are analyzed numerically.

Nomenclature

A_x	horizontal oscillation amplitude	(m)
A_y	vertical oscillation amplitude	(m)
D_p	particle diameter	(m)
e	coefficient of restitution between particle and substrate	(-)
F_g	gravity force	(N)
F_{dx}	horizontal component of drag force	(N)
F_{dy}	vertical component of drag force	(N)
f	vibration frequency	(Hz)
m_p	mass of particle	(kg)
t	time	(s)
v_{px}	horizontal velocity of particle	(m/s)
v_{py}	vertical velocity of particle	(m/s)
v_{sx}	horizontal velocity of substrate	(m/s)
v_{sy}	vertical velocity of substrate	(m/s)
x_p	horizontal displacement of particle	(m)
x_s	horizontal displacement of substrate	(m)
y_p	vertical displacement of particle	(m)
y_s	vertical displacement of substrate	(m)
μ	coefficient of friction between particle and substrate	(-)
ρ_p	particle density	(kg/m^3)
ω	vibration angular velocity ($=2\pi f$)	(rad/s)

Subscripts

1	before collision
2	after collision

2. Numerical model

A horizontal substrate is driven by an oblique linear oscillation. The x and y coordinates are defined by setting the x axis and y axis tangential and vertical (upward) to the substrate, respectively. The horizontal displacement x_s and vertical displacement y_s of the oscillating substrate are given by

$$x_s = A_x \sin \omega t \tag{1}$$

and

$$y_s = A_y \sin \omega t \tag{2}$$

where A_x and A_y are the horizontal and vertical oscillation amplitudes, respectively; ω is the angular velocity; and t is the time. The motion equations of particles during flight are represented by

$$m_p \frac{dv_{px}}{dt} = F_{dx} \tag{3}$$

and

$$m_p \frac{dv_{py}}{dt} = F_g + F_{dy} \tag{4}$$

where m_p is the mass of a particle, v_p is the velocity of a particle, F_g is the force of gravity, and F_d is the drag force. For larger particles, F_d is negligible compared to F_g. The relative vertical velocities between the particle and the substrate before and after a collision are related by the coefficient of restitution e, i.e.,

$$v_{py2} - v_{sy2} = -e(v_{py1} - v_{sy1}) \tag{5}$$

where v_{sy} is the vertical velocity of the substrate. The subscripts “1” and “2” represent before and after a collision, respectively. If it is assumed that the mass of the substrate is much greater than that of the particle, the velocity of the substrate after the collision is equivalent to that before the collision because the collision has a negligible effect on the motion of the substrate:

$$v_{sy2} = v_{sy1} \tag{6}$$

The horizontal component J_x and vertical component J_y of the impulse exerted by the substrate on the particle during the collision are represented by

$$J_x = m_p (v_{px2} - v_{px1}) \tag{7}$$

and

$$J_y = m_p (v_{py2} - v_{py1}) \tag{8}$$

In collisions that involve sliding, if it is assumed that the sliding is described by Coulomb’s law of friction, the horizontal and vertical components of the impulse are related by the coefficient of friction, μ:

$$J_x = -\mu J_y \qquad (v_{px1} > v_{sx1}) \tag{9.1}$$

$$J_x = 0 \qquad (v_{px1} = v_{sx1}) \tag{9.2}$$

$$J_x = +\mu J_y \qquad (v_{px1} < v_{sx1}) \tag{9.3}$$

The horizontal rebound velocity of a particle just after collision, v_{px2}, is obtained by substituting Eqs. (5)–(8) into Eqs. (9.1)–(9.3):

$$v_{px2} = v_{px1} - \mu(v_{sy} - v_{py1})(e+1) \qquad (v_{px1} > v_{sx1}) \tag{10.1}$$

$$v_{px2} = v_{px1} \qquad (v_{px1} = v_{sx1}) \tag{10.2}$$

$$v_{px2} = v_{px1} + \mu(v_{sy} - v_{py1})(e+1) \qquad (v_{px1} < v_{sx1}) \tag{10.3}$$

Here, we need to consider a condition that the tangential component of the relative velocity cannot reverse its direction during the collision because the sliding stops when this component decreases to zero. Therefore, it is physically impossible that v_{px2} calculated from Eq. (10.1) is lower than v_{sx1} and v_{px2} calculated from Eq. (10.3) is higher than v_{sx1}. In the case, the following equation is employed

$$v_{px2} = v_{sx1} \quad . \tag{11}$$

The two-dimensional movements of the particles and the substrate can be calculated using Eqs. (1)–(6), (10) and (11). To examine the effects of the restitution and drag force on the transport velocity, the movements of three particles with different coefficients of restitution and diameters were calculated (particle A: D_p = 500 μm, e = 0.9; particle B: D_p = 500 μm, e = 0.5; and particle C: D_p = 50 μm, e = 0.9). The calculation conditions used for the simulations are listed in Table 1.

Table 1. Calculation conditions.

Parameter	Value
Horizontal amplitude A_x (μm)	50
Vertical amplitude A_y (μm)	50
Frequency f (Hz)	300
Particle diameter D_p (μm)	50, 500
Particle density ρ_p (kg/m^3)	5000
Coefficient of restitution e (-)	0.5, 0.9
Coefficient of friction μ (-)	0.1

3. Results

Fig.1 shows the horizontal displacements as a function of the elapsed time for each particle. The displacements of all the particles increase with time, and particle A has significant fluctuations. Particles A, B, and C have average transport velocities over the time interval of 0–10 s: 34.5, 68.9, and 21.0 mm/s, respectively. Thus, a particle with a higher restitution and smaller diameter has a smaller transport velocity. To clarify the mechanism for the difference in the transport velocity, the particle trajectories and rebound horizontal velocity distributions after collisions are analyzed.

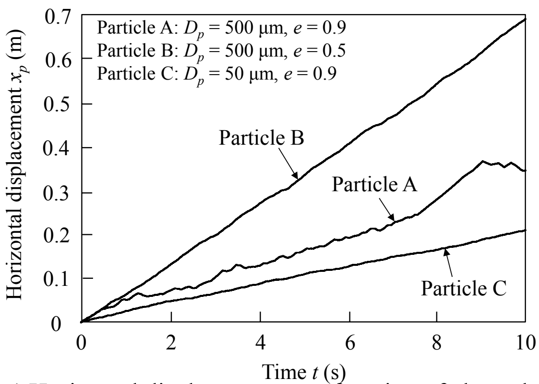

Fig.1 Horizontal displacement as a function of elapsed time.

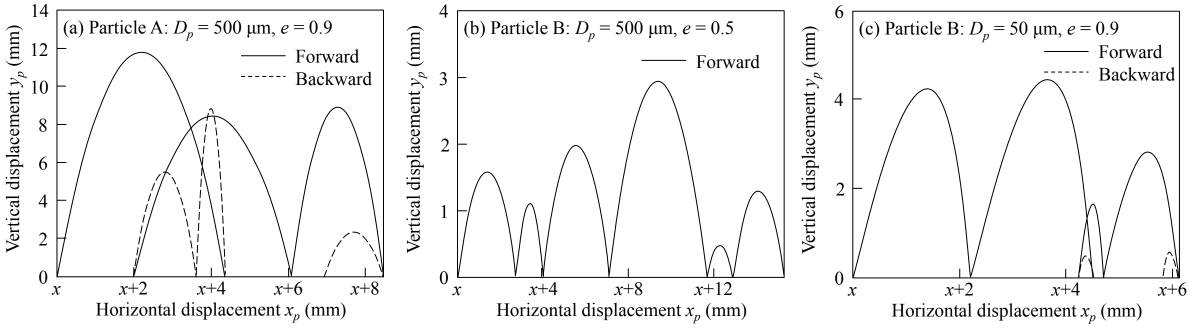

Fig.2 Particle trajectory on obliquely oscillating plate.

Figs.2 (a)–(c) show the trajectories of particles A, B, and C, respectively. From Figs.2 (a) and (c), we can observe that particles A and C bounce forward and backward repeatedly. Particle B only bounces forward (Fig.2 (b)). In Fig.2 (a), the bounce height range of particle A is up to 12 mm, and the horizontal distances of the bounces have a range of –1.7 to 4.3 mm. Particle B saltates slightly in a height range of 0–3 mm, and the horizontal distances of the bounces have a range of 1.2–4.5 mm (Fig.2 (b)). Particle C reaches a maximum height of 4.5 mm, and the horizontal distances have a range of –0.3 to 2.4 mm (Fig.2 (c)).

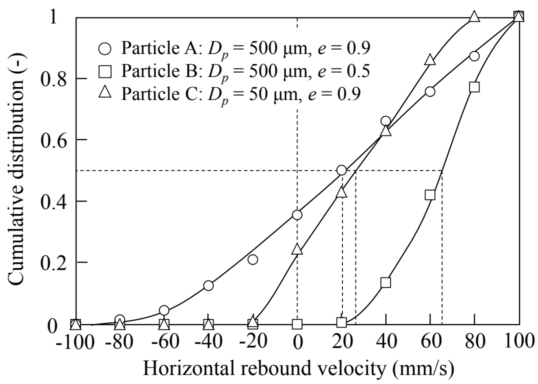

Fig.3 Number-based distribution of horizontal rebound velocity.

Fig.3 shows number-based distributions of the horizontal rebound velocities calculated from 200 collisions. The positive and negative horizontal rebound velocities mean forward and backward bounces, respectively. Particles A, B, and C have median velocities of 20 mm/s, 66 mm/s, and 27 mm/s, respectively. The rebound velocity distributions of particles A, B, and C have ranges of –100 to 100 mm/s, 20 to 100 mm/s, and –20 to 80 mm/s, respectively. The ratios of the backward bounces to all the bounces for particles A, B, and C are 0.36, 0, and 0.25, respectively.

4. Discussion

A higher bounce height increases the vertical component of the impulse J_y exerted on a particle during a collision. As seen from Eqs. (9.1) and (9.3), an increase in the vertical impulse increases the magnitude of the horizontal component of the impulse J_x, which induces a drastic change in the horizontal velocity from the collision. First, we compare particle A with particle B. As seen in a comparison between Fig.2 (a) and Fig.2 (b), the bounce height of particle A is much higher than that of particle B because of its higher restitution. Therefore, particle A has a higher ratio of backward bounces and a lower horizontal rebound velocity (Fig.3). As a result, in Fig.1, the transport velocity of particle A is lower than that of particle B. Second, we compare particle A with particle C. As seen in a comparison between Fig.2 (a) and Fig.2 (c), the bounce height of particle C is lower than that of particle A because of the greater effect of the drag force during flights. Therefore, particle C has a lower ratio of backward bounces and a higher median horizontal rebound velocity (Fig.3). However, in Fig.1, the transport velocity of particle C is lower than that of particle A. This can be explained as follows. While the horizontal velocity of particle A remains at the rebound velocity during flight between successive collisions, the horizontal velocity of particle C decreases as a result of the drag force.

5. Conclusions

To examine the effects of the restitution and fluid resistance on the horizontal transport velocity, two-dimensional movements of fine particles on an obliquely oscillating plate were simulated using a mass-point model that considered the gravity, fluid resistance, restitution, and friction. The results showed that a particle with a higher restitution and smaller diameter had a smaller transport velocity. This could be explained by discussing the impulse exerted on the particle during a collision and the effect of the drag force during flight.

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